Ενότητα 1 — Λύσεις

1) $(x+5)^2 = x^2+10x+25$

$(x-2)^2 = x^2-4x+4$

$(7-2x)^2 = 49-28x+4x^2 = 4x^2-28x+49$

$(4a+y)^2 = 16a^2+8ay+y^2$

$(x+4)(x-4) = x^2-16$

$(3x-4)(3x+4) = 9x^2-y^2$

$(2\beta+5)(5-2\beta) = (5+2\beta)(5-2\beta) = 25-4\beta^2$

$(x+2)^3 = x^3+3\cdot x^2\cdot 2+3\cdot x\cdot 2^2+2^3 = x^3+6x^2+12x+8$

$(x-\frac{y}{2})(x+\frac{y}{2}) = x^2-\frac{y^2}{4}$

$(\frac{2}{x}+\frac{x}{2})^2 = \frac{4}{x^2}+2\cdot\frac{2}{x}\cdot\frac{x}{2}+\frac{x^2}{4} = \frac{4}{x^2}+2+\frac{x^2}{4}$

$(x+10)^2(x-10)^2 = [(x+10)(x-10)]^2 = (x^2-100)^2 = x^2-200x+10000$

$(\sqrt{\beta}-3)^2 = \beta-6\sqrt{\beta}+9$

$(x^2+4)(x-2)(x+2) = (x^2+4)(x^2-4) = x^4-16$

$(-5+2x)^2 = (2x-5)^2 = 4x^2-20x+25$

$(\frac{3}{5}x^3y^2-5\beta^3)(\frac{3}{5}x^3y^2+5\beta^3) = \frac{9}{25}x^6y^4-25\beta^6$

$(3a-\beta)(9a^2+\beta^2)(3a+\beta) = (9a^2-\beta^2)(9a^2+\beta^2) = 81a^4+\beta^4$

$(x-\sqrt{2})^2 = x^2-2\sqrt{2}x+2$

$(\sqrt{x}+\frac{1}{\sqrt{y}})^2 = x+\frac{2\sqrt{x}}{\sqrt{y}}+\frac{1}{y} = x+2\sqrt{\frac{x}{y}}+\frac{1}{y}$

2) Κάθε ισότητα που περιέχει μεταβλητές είναι αληθεύει για όλες τις τιμές των μεταβλητών αυτών.

3) Ταυτότητες είναι οι: $0x=0$, $\beta^2\beta=\beta^3$, $(x+3)^2=x^2+6x+9$

4) Ορθές είναι: (β), (ε) και το (στ)

5) $(a+3)^2 = a^2+6a+9$ $\quad (\sqrt{2}y-4)^2 = 2y^2-8\sqrt{2}y+16$

$(\beta^3-\beta^4)(\beta^3+\beta^4) = \beta^6+\beta^8$ $\quad (-a-\frac{3}{\beta})^2 = a^2+\frac{6a}{\beta}+\frac{9}{\beta^2}$

6) $P(x) = (x-3)^2+(3x+1)^2-10(x-1)(x+1) = x^2-6x+9+9x^2+6x+1-10x^2+10 = 20$ σταθερό

7) $(x-4)^2+(2x+5)^2 = x^2-8x+16+4x^2+20x+25 = 5x^2+12x+41$

$(x^2-1)^2-(x^2-3)(x^2+3) = x^4-2x^2+1-x^4+9 = -2x^2+10$

$(\gamma^2+\gamma)^3-(\gamma^2-\gamma)^3 = \gamma^6+3\gamma^5+3\gamma^4+\gamma^3-\gamma^6+3\gamma^5-3\gamma^4+\gamma^3 = 6\gamma^5+2\gamma^3$

$(4x-1)^3-x(8x+1)(8x-1) = 64x^3-48x^2+12x-1-64x^3+x = -48x^2+13x-1$

$(2x^2+2x)^2+(2x+1)^2 = 4x^4+8x^3+4x^2+4x^2+4x+1 = 4x^4+8x^3+8x^2+4x+1$

8) $(x-2)(x+2)+(x-2)^2+2 = x^2-4+x^2-4x+4+2 = 2x^2-4x+2 = 2(x^2-2x+1) = 2(x-1)^2$

$(a+1)^3-(a+1)(a-1)^2-4a^2 = (a+1)^3-(a^2-1)(a-1)-4a^2 = a^3+3a^2+3a+1-a^3+a^2+a-1-4a^2 = 4a$

$(w-2)^3-w(w-5)^2+w+17 = w^3-6w^2+12w-8-w(w^2-10w+25)+w+17 =$

$= w^3-6w^2+12w-8-w^3+10w^2-25w+w+17 = 4w^2-12w+9 = (2w-3)^2$

$\begin{cases} (a-b)^3-a(a-3b)^2+6ab^2 = a^3-3a^2b+3ab^2+b^3-a(a^2-6ab+9b^2)+6ab^2 = a^3-3a^2b+3ab^2+b^3-a^3+6a^2b-9ab^2+6ab^2 \\ b(3a^2-b^2) = 3a^2b-b^3 \end{cases}$

$= 3a^2b-6ab^2+6b^2+b^3 = 3a^2b+b^3 \Big\} A_H = B_H$

$\frac{b^2+\gamma^2-(b-\gamma)^2}{2} = \frac{b^2+\gamma^2-b^2+2b\gamma-\gamma^2}{2} = \frac{2b\gamma}{2} = b\gamma$

9) $\left(a+\frac{5}{a}\right)^2-\left(a-\frac{5}{a}\right)^2 = a^2+2a\frac{5}{a}+\frac{25}{a^2}-a^2+2a\frac{5}{a}-\frac{25}{a^2} = 10+10 = 20$

$Z = \left(2020+\frac{1}{404}\right)^2-\left(2020-\frac{1}{404}\right)^2 = \left(2020+\frac{5}{2020}\right)^2-\left(2020-\frac{5}{2020}\right)^2 = 20$ αφου $|a|$ εφαπ. για $a=2020$

10) $x = 3+\sqrt{5}$ $y = 3-\sqrt{5}$ $\Rightarrow x+y = 3+\sqrt{5}+3-\sqrt{5} = 6$

$xy = (3+\sqrt{5})(3-\sqrt{5}) = 9-5 = 4$

$x^2-y^2 = (x-y)(x+y) = [(3+\sqrt{5})-(3-\sqrt{5})][(3+\sqrt{5})+(3-\sqrt{5})] = (3+\sqrt{5}-3+\sqrt{5})(3+\sqrt{5}+3-\sqrt{5}) = 2\sqrt{5}\cdot 6 = 12\sqrt{5}$

$x^2+y^2 = (x+y)^2-2xy = 6^2-2\cdot 4 = 36-8 = 28$

x^3+y^3 α) $x^3+3x^2y+3xy^2+y^3 = (x+y)^3 \Rightarrow x^3+3xy(x+y)+y^3 = (x+y)^3 \Rightarrow x^3+y^3 = (x+y)^3-3xy(x+y)$

$\Rightarrow x^3+y^3 = 6^3-3\cdot 4\cdot 6 = 216-72 = 144$

β) $x^3+y^3 = (x+y)(x^2-xy+y^2) = 6\cdot(28-4) = 6\cdot 24 = 144$

11) $\Pi_{ABΓΔ} = 2(AB)+2(BΓ) = 2(4x+3)^2+2(3x-1)^2 = 2(16x^2+24x+9)+2(9x^2-6x+1) =$

$= 32x^2+48x+18+18x^2-12x+2 = 50x^2+36x+20$ cm

$\Pi_{EZHΘ} = 2(HZ)+2(EZ) = 2(18x+11)+2(5x-1)(5x+1) = 36x+22+50x^2-2 = 50x^2+36x+20$ cm $\Big\} \Rightarrow$

$\Pi_{ABΓΔ} = \Pi_{EZHΘ}$ με $\Pi_{ABΓΔ} = \Pi(x) = 50x^2+36x+20$ cm

$50x^2+36x+20 = 106 \Rightarrow 50x^2+36x-86 = 0 \Rightarrow 25x^2+18x-43 = 0$

$x_{12} = \frac{-18\pm\sqrt{18^2-4\cdot 25\cdot(-43)}}{2\cdot 25} = \frac{-18\pm\sqrt{324+4300}}{50} = \frac{-18\pm\sqrt{4624}}{50} = \frac{-18\pm 68}{50} \Rightarrow \frac{50}{50} = 1$ ή $-\frac{86}{50} = -\frac{43}{25}$ Αφορ

Αρα $x = 1$

HZ, EZ ⊥ΘΗ

12) $16+(b^2-4)(b^2+4) = 16+b^4-16 = b^4$

$\sqrt{\sqrt{16+96\cdot 104}} = \sqrt{\sqrt{16+(10^2-4)(10^2+4)}} = \sqrt{\sqrt{10^4}} = \sqrt{10^2} = 10$ αφου $|a|$ εφαπ. για $b=10$

13) $A = x^3+4x^2+4x = x(x^2+4x+4) = x(x+2)^2$ ή $A = x(x+2)^2 = x(x^2+4x+4) = x^3+4x^2+4x$

$B = x^2-4 = (x-2)(x+2)$ $\Gamma = x^2-2x = x(x-2)$

$\frac{1}{A}+\frac{2}{B} = \frac{1}{\Gamma} \Rightarrow \frac{x-2}{x(x+2)^2}+\frac{x(x+2)}{(x-2)(x+2)} = \frac{(x+2)^2}{x(x-2)}$ $x\neq 0,\ x\neq 2,\ x\neq -2$

$\Rightarrow (x-2)+2x(x+2) = (x+2)^2 \Rightarrow x-2+2x^2+4x-x^2-4x-4 = 0 \Rightarrow$

$\Rightarrow x^2+x-6 = 0 \Rightarrow (x+3)(x-2) = 0$

$x+3=0$ | $x-2=0$

$x=-3$ Δεκτή | $x=2$ Απορ

14| $2^x = 12$ και $6^y = 12$

α) $\Rightarrow 2^x = 6^y$ ισοδύναμη με 2^y

$\Rightarrow 2^x \cdot 2^y = 6^y \cdot 2^y \Rightarrow$

$\Rightarrow 2^{x+y} = (12)^y$ $\quad (a\cdot b)^v = a^v b^v$, $a^v \cdot a^\mu = a^{v+\mu}$

$\Rightarrow 2^{x+y} = (2^x)^y$ $\quad 2^x = 12$

$\Rightarrow 2^{x+y} = 2^{xy}$ $\quad (a^v)^\mu = a^{v\cdot\mu}$

$\Rightarrow \boxed{x+y = xy}$

β| $A = \dfrac{\frac{x+y}{x-y} - \frac{x-y}{x+y}}{\frac{1}{x-y}} = \dfrac{\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}}{\frac{1}{x-y}} =$ $\quad x \neq \pm y$

$= \dfrac{\frac{(x+y+x-y)(x+y-x+y)}{(x-y)(x+y)}}{\frac{1}{x-y}} = \dfrac{(x-y)(2x)\cdot(2y)}{(x-y)(x+y)} = \dfrac{4xy}{x+y} \overset{(a)}{=} \dfrac{4xy}{xy} = \underline{4}$

γ| $B = \left(\frac{x}{y} + \frac{y}{x} - 2\right) : \left(\frac{1}{y} - \frac{1}{x}\right) = \dfrac{x^2+y^2-2xy}{xy} : \dfrac{x-y}{xy} = \dfrac{(x-y)^2}{xy} \cdot \dfrac{xy}{x-y} = x-y$

$\Gamma = \left(\frac{1}{x} + \frac{1}{y}\right) \cdot xy = \dfrac{y+x}{xy} \cdot xy = x+y$

δ) $5B > A\Gamma \Rightarrow 5(x-y) > 4\cdot(x+y) \Rightarrow 5x-5y > 4x+4y \Rightarrow$

$\Rightarrow 5x-4x > 5y+4y \Rightarrow \underline{x > 9y}$

ε) $x+y = xy = 5$ βλ

$\dfrac{x}{y} + \dfrac{y}{x} = \dfrac{x^2+y^2}{xy} = \dfrac{(x+y)^2-2xy}{xy} = \dfrac{5^2-2\cdot5}{5} = \dfrac{25-10}{5} = \underline{5}$

$\dfrac{x}{y^2} + \dfrac{y}{x^2} = \dfrac{x^3+y^3}{x^2y^2} = \dfrac{(x+y)^3-3xy(x+y)}{x^2y^2} = \dfrac{5^3-3\cdot5\cdot5}{5^2} = \dfrac{125-75}{25} = \dfrac{50}{25} = \underline{2}$