

Ερώληα 2 - Πύσεις

1) $4x-12=4(x-3)$

$5x+5y-15=5(x+y-3)$

$5xy^3+10y=5y(xy^2+1)$

$y^2-16=(y-4)(y+4)$

$4x^2-16=4(x^2-4)=4(x-2)(x+2)$

$x^3+1000=x^3+10^3=(x+10)(x^2-10x+100)$

$x^2-6x+8=(x-4)(x-2)$

$x^2+9x+18=(x+3)(x+6)$

$3x^2-27=3(x^2-9)=3(x-3)(x+3)$

$x^3+27-3x^2-9x=(x+3)(x^2-3x+9)-3x(x+3)=(x+3)(x^2-3x+9-3x)=(x+3)(x^2-6x+9)=(x+3)(x-3)^2$

$xy+5y-x^2-2x+15=y(x+5)-(x^2+2x-15)=y(x+5)-(x+5)(x-3)=(x+5)(y-x+3)$

$w^3+w^2-36=w^3-27+w^2-9=(w-3)(w^2+3w+9)+(w-3)(w+3)=(w-3)(w^2+3w+9+w+3)=(w-3)(w^2+4w+12)=\cancel{(w-3)(w+4)}$

$(2x-6)(x^2-1)-(4x-12)(x-1)^2=2(x-3)(x-1)(x+1)-4(x-3)(x-1)^2=2(x-3)(x-1)(x+1-2(x-1))=2(x-3)(x-1)(-1-x)=-2(x-3)(x-1)(x+1)$

2) $3x^2-2x-1=0$ $a=3, b=-2, c=-1$

$\Delta=b^2-4ac \Rightarrow \Delta=(-2)^2-4 \cdot 3 \cdot (-1)=4+12=16 \Rightarrow x_{1,2}=\frac{-b \pm \sqrt{\Delta}}{2a}=\frac{2 \pm \sqrt{16}}{6}=\frac{2 \pm 4}{6} \rightarrow \frac{1}{3}$

$4x^2-5x+1=0$
 $\Delta=25-16=9 \Rightarrow x_{1,2}=\frac{5 \pm \sqrt{9}}{8}=\frac{5 \pm 3}{8} \rightarrow \frac{1}{4}$

$2y^2+7y-4=0$
 $\Delta=49+32=81 \Rightarrow y_{1,2}=\frac{-7 \pm \sqrt{81}}{4}=\frac{-7 \pm 9}{4} \rightarrow \frac{1}{2}$

$3x^2=10x-3 \Rightarrow 3x^2-10x+3=0$
 $\Delta=100-36=64 \Rightarrow x_{1,2}=\frac{10 \pm \sqrt{64}}{6}=\frac{10 \pm 8}{6} \rightarrow \frac{1}{3}$

$x(2x+1)=1 \Rightarrow 2x^2+x-1=0$
 $\Delta=1+8=9 \Rightarrow x_{1,2}=\frac{-1 \pm \sqrt{9}}{4}=\frac{-1 \pm 3}{4} \rightarrow -1$

$2x^2=9x \Rightarrow 2x^2-9x=0 \Rightarrow x(2x-9)=0 \Rightarrow x=0$ ή $2x-9=0 \Rightarrow x=\frac{9}{2}$

$w(5-w)=4 \Rightarrow 5w-w^2=4 \Rightarrow w^2-5w+4=0 \Rightarrow (w-1)(w-4)=0 \Rightarrow w=1$ ή $w=4$

$15x^2-25x+10=0 \Rightarrow 5(3x^2-5x+2)=0 \Rightarrow 3x^2-5x+2=0$
 $\Delta=25-24=1 \Rightarrow x_{1,2}=\frac{5 \pm \sqrt{1}}{6}=\frac{5 \pm 1}{6} \rightarrow \frac{2}{3}$

$3x^2-2x-1=0$
 $\Delta=4+12=16 \Rightarrow x_{1,2}=\frac{2 \pm \sqrt{16}}{6}=\frac{2 \pm 4}{6} \rightarrow -\frac{1}{3}$

$3(x^2-x)=(x-1)(x+1) \Rightarrow 3x(x-1)-(x-1)(x+1)=0 \Rightarrow (x-1)(3x-x-1)=0 \Rightarrow (x-1)(2x-1)=0 \Rightarrow x=1$ ή $x=\frac{1}{2}$

$2x^3+3x^2=5x \Rightarrow 2x^3+3x^2-5x=0 \Rightarrow x(2x^2+3x-5)=0 \Rightarrow x=0$ ή $2x^2+3x-5=0$

$\Delta=9+40=49 \Rightarrow x_{1,2}=\frac{-3 \pm \sqrt{49}}{4}=\frac{-3 \pm 7}{4} \rightarrow -\frac{5}{2}$

$$3) \left(\frac{4}{x^2-4} - \frac{2}{x^2+2x} - \frac{1}{2-x} \right) \div \frac{x+2}{x^3-x^2-2x} =$$

$$= \left[\frac{4}{(x-2)(x+2)} - \frac{2}{x(x+2)} + \frac{1}{x-2} \right] \div \frac{x+2}{x(x^2-x-2)} =$$

$$= \left[\frac{4}{(x-2)(x+2)} - \frac{2}{x(x+2)} + \frac{1}{x-2} \right] \div \frac{x+2}{x(x-2)(x+1)} =$$

$$= \frac{4x - 2x + 4 + x^2 + 2x}{x(x-2)(x+2)} \cdot \frac{x(x-2)(x+1)}{x+2} =$$

$$= \frac{x^2 + 4x + 4}{(x+2)^2} = \frac{(x+2)^2}{(x+2)^2} = 1$$

$$E(KT) = x(x-2)(x+2)$$

$$x \neq 0 \quad x \neq 2 \quad x \neq -2$$

$$a) \text{ κα } x(x-2)(x+1) \neq 0 \Rightarrow \begin{matrix} x \neq 0 \\ x \neq -1 \\ x \neq 2 \end{matrix}$$

$$\delta \text{ ι α ρ ε ο ς } \Rightarrow x+2 \neq 0 \Rightarrow x \neq -2$$

$$\text{Άρα } x \neq 0, x \neq 2, x \neq -2, x \neq -1$$

$$4) \frac{y^2-3y+2}{y^2-1} \div \frac{5y-10}{y^2+4} = \frac{(y-2)(y+1)}{(y-1)(y+1)} \cdot \frac{y(y+1)}{5(y-2)} = \frac{y}{5}$$

$$\frac{x^2-5x}{x^2-7x+10} \cdot \frac{x-2}{x^2+x} = \frac{x(x-5)}{(x-5)(x-2)} \cdot \frac{x-2}{x(x+1)} = \frac{1}{x+1}$$

$$\left(\frac{x-5}{x-4} \right) \div \frac{10-2x}{x^2-16} = \frac{x^2-4x-5}{x-4} \cdot \frac{x^2-16}{10-2x} = \frac{(x-5)(x+1)}{x-4} \cdot \frac{(x-4)(x+4)}{-2(x-5)} = \frac{(x+1)(x+4)}{2}$$

$$\frac{x+3}{x-3} - \frac{5}{x} + \frac{18}{3x-x^2} = \frac{x}{x-3} - \frac{5}{x} + \frac{18}{x(x-3)} = \frac{x^2+3x-5x+15-18}{x(x-3)} = \frac{x^2-2x-3}{x(x-3)}$$

$$= \frac{(x-3)(x+1)}{x(x-3)} = \frac{x+1}{x}$$

$$\frac{y^2-4}{y^2+4y+4} \div \left(\frac{5}{y+2} - \frac{3}{y} \right) = \frac{y^2-4}{y^2+4y+4} \cdot \frac{y(y+2)}{y(y+2)-3(y+2)} = \frac{(y-2)(y+2)}{(y+2)^2} \cdot \frac{y(y+2)}{2y-6} = \frac{y(y-2)}{2(y-3)}$$

$$\frac{2}{x^2+3x} - \frac{4}{x^2-9} - \frac{1}{3-x} = \frac{2}{x(x+3)} - \frac{4}{(x-3)(x+3)} + \frac{1}{x-3} = \frac{2x-6-4x+x^2+3x}{x(x-3)(x+3)} = \frac{x^2+x-6}{x(x-3)(x+3)}$$

$$= \frac{(x+3)(x-2)}{x(x-3)(x+3)} = \frac{x-2}{x(x-3)}$$

$$\left(\frac{1}{1} + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \right) \div \frac{x^4-1}{x^4-x^3} = \frac{x^4-1}{x^3} \cdot \frac{x^3(x-1)}{(x^2-1)(x^2+1)} = \frac{[x^2(x+1)+(x+1)] \cdot (x-1)}{(x-1)(x+1)(x^2+1)}$$

$$= \frac{(x+1)(x^2+1)}{(x+1)(x^2+1)} = 1$$

$$\left(\frac{3}{w-2} - \frac{1}{w+2} \right) \div \frac{w^2+8w+16}{w^3-4w} = \frac{3w+6-w+2}{(w-2)(w+2)} \cdot \frac{w(w^2-4)}{(w+4)^2} = \frac{2w+8}{(w-2)(w+2)} \cdot \frac{w(w-2)(w+2)}{(w+4)^2}$$

$$= \frac{2(w+4)}{(w+4)^2} = \frac{2}{w+4}$$

$$\frac{9x^2-12x+4}{2x-6x} \circ \left(\frac{x}{x^2-3x} - \frac{8-x}{2x^2-18} \right) = \frac{9x^2-12x+4}{2x^2-6x} \circ \left(\frac{x}{x(x-3)} - \frac{8-x}{2(x^2-9)} \right) =$$

$$= \frac{9x^2-12x+4}{2x^2-6x} \circ \left[\frac{\frac{2(x+3)}{x}}{x(x-3)} - \frac{8-x}{2(x-3)(x+3)} \right] = \frac{(3x+2)^2}{2x(x-3)} \circ \frac{2x^2+6x-8x+x^2}{2x(x-3)(x+3)} =$$

$$= \frac{(3x-2)^2}{2x(x-3)} \circ \frac{2x(x-3)(x+3)}{3x^2+6x-8x} = \frac{(3x-2)^2 \cdot (x+3)}{x(3x-2)} = \frac{(3x-2)(x+3)}{x}$$

$$\frac{(x^3+2x^2)/(x-1)}{x^2-2x+1} \circ \frac{x^2-4}{x^2-3x+2} = \frac{x^2(x+2)(x+1)}{(x-1)^2} \circ \frac{(x-2)(x+1)}{(x-2)(x+2)} = x^2$$

$$5) \frac{\frac{a^2}{2} - \frac{4}{a}}{2a^2-4a} + 2a = \frac{\frac{a^3-8}{2a}}{4a^2} + 2a = \frac{(a-2)(a^2+2a+4) \cdot 4a^x}{2a \cdot 2a(a-2)} + 2a =$$

$$= a^2 + 2a + 4 + 2a = a^2 + 4a + 4 = (a+2)^2$$

$$\left(\frac{x}{y} - \frac{2}{1} + \frac{y}{x} \right) \circ \left(\frac{x}{xy+y^2} - \frac{y}{x^2+xy} \right) = \frac{x^2-2xy+y^2}{xy} \circ \left(\frac{x}{y(x+y)} - \frac{y}{x(x+y)} \right) =$$

$$= \frac{(x-y)^2}{xy} \circ \frac{x^2-y^2}{xy(x+y)} = \frac{(x-y)^2}{xy} \circ \frac{(x-y)(x+y)}{(x-y)(x+y)} = x-y$$

$$(3a+2b)^2 - 5(a-2b)(a+2b) - 3b(8a+5b) =$$

$$= 9a^2 + 12ab + 4b^2 - 5(a^2 - 4b^2) - 24ab - 15b^2 =$$

$$= 9a^2 + 12ab + 4b^2 - 5a^2 + 20b^2 - 24ab - 15b^2 =$$

$$= 4a^2 - 12ab + 9b^2 = (2a-3b)^2$$

$$\frac{a + \frac{b^2}{a} - 2b}{1 - \frac{b}{a}} = \frac{\frac{a^2+b^2-2ab}{a}}{\frac{a-b}{a}} = \frac{(a-b)^2 \cdot a}{a \cdot (a-b)} = a-b$$

$$\left(\frac{6x}{x^2-9} + \frac{2x}{x+3} \right) \circ \frac{x^2+6x+9}{2x^2} = \left[\frac{6x}{(x-3)(x+3)} + \frac{2x}{x+3} \right] \circ \frac{(x+3)^2}{2x^2} = \frac{6x+2x^2-6x}{(x-3)(x+3)} \circ \frac{(x+3)^2}{2x^2} =$$

$$= \frac{2x^2}{x-3} \circ \frac{x+3}{2x^2} = \frac{x+3}{x-3}$$

$$6) \frac{2w-1}{w+3} + \frac{w-4}{2-w} = \frac{35}{w^2+w-6} \Rightarrow \frac{2w-1}{w+3} - \frac{w-4}{w-2} = \frac{35}{(w+3)(w-2)}$$

EKΠ = (w+3)(w-2)
w ≠ -3, w ≠ 2

$$\Rightarrow (2w-1)(w-2) - (w+3)(w-4) = 35 \Rightarrow$$

$$\Rightarrow 2w^2 - 4w - w + 2 - w^2 + 4w - 3w + 12 = 35 \Rightarrow$$

$$\Rightarrow w^2 - 4w - 21 = 0 \Rightarrow$$

$$(w-7)(w+3) = 0 \Rightarrow w=7 \text{ veya } w=-3 \text{ Awp}$$

$$\frac{4x+4}{x^2+2x-15} - \frac{2}{x-3} = \frac{x+4}{x^2+5x} \Rightarrow$$

$$EKT = x(x-3)(x+5)$$

$$x \neq 0, x \neq 3, x \neq -5$$

$$\frac{4x+4}{(x+5)(x-3)} - \frac{\frac{x(x+5)}{2}}{(x-3)} = \frac{\frac{x-3}{x+4}}{x(x+5)} \Rightarrow$$

$$x(4x+4) - 2x(x+5) = (x-3)(x+4) \Rightarrow$$

$$4x^2+4x - 2x^2-10x = x^2+4x-3x-12 \Rightarrow$$

$$4x^2+4x - 2x^2-10x - x^2-4x+3x+12 \Rightarrow 0 \Rightarrow$$

$$x^2 - 7x + 12 = 0 \Rightarrow (x-4)(x-3) = 0 \Rightarrow x=4 \text{ oder } x=3 \text{ Awp}$$

$$\frac{x-7}{x^2-5x+4} - \frac{1}{4-x} = \frac{x-3}{x^2-x} \Rightarrow \frac{x-7}{(x-4)(x-1)} + \frac{1}{x-4} = \frac{x-3}{x(x-1)} \Rightarrow$$

$$x \neq 1, x \neq 0, x \neq 4$$

$$\Rightarrow x^2-7x+x-x = x^2-4x-3x+12 \Rightarrow x^2-7x-x+4x+3x-12=0$$

$$\Rightarrow x^2-x-12=0 \Rightarrow (x-4)(x+3)=0 \Rightarrow x=4 \text{ Awp } x=-3 \text{ oder } 4$$

$$\frac{7(x+1)}{x^2-x-12} + \frac{2x-3}{4-x} = \frac{1-3x}{x+3} \Rightarrow \frac{7x+7}{(x-4)(x+3)} + \frac{2x-3}{x-4} = \frac{1-3x}{x+3} \Rightarrow x \neq 4, x \neq -3$$

$$\Rightarrow 7x+7 - (x+3)(2x-3) = (x-4)(1-3x) \Rightarrow 7x+7 - 2x^2+3x-6x+9 = x-3x^2-4+12x \Rightarrow$$

$$\Rightarrow 7x+7 - 2x^2+3x-6x+9 - x+3x^2+4-12x = 0 \Rightarrow x^2-9x+20=0 \Rightarrow (x-5)(x-4)=0$$

$$\Rightarrow x=5 \text{ oder } x=4 \text{ Awp}$$

$$\frac{2}{x-3} + \frac{x}{2x-4} = \frac{2}{x^2-5x+6} \Rightarrow \frac{2(x-2)}{x-3} + \frac{x-3}{2(x-2)} = \frac{2}{(x-2)(x-3)} \Rightarrow x \neq 2 \text{ oder } x \neq 3$$

$$\Rightarrow 4(x-2) + x(x-3) = 4 \Rightarrow 4x-8 + x^2-3x-4 = 0 \Rightarrow$$

$$\Rightarrow x^2+x-12=0 \Rightarrow (x+4)(x-3)=0 \Rightarrow x=-4 \text{ oder } x=3 \text{ Awp}$$

$$\frac{x+3}{x-3} + \frac{3}{3x-x^2} = \frac{1}{x} \Rightarrow \frac{x+3}{x-3} + \frac{1}{x(x-3)} = \frac{1}{x} \Rightarrow$$

$$x \neq 0 \text{ oder } x=3$$

$$\Rightarrow x^2+3x+3 = x-3 \Rightarrow x^2+2x+6=0 \Rightarrow x(x+2)=0 \Rightarrow x=0 \text{ Awp } x=-2 \text{ oder } 4$$

$$7a) A = \frac{x + \frac{2x}{x-2}}{1 + \frac{4}{x^2-2x}} = \frac{\frac{x^2-2x+2x}{x-2}}{\frac{x^2-4+4}{x^2-2x}} = \frac{x^2(x-2)(x+2)}{(x-4)x^2} = x+2$$

$$B = \frac{x^3-x^2-x+1}{2x^3-14x^2-16x} = \frac{x^2(x-1)-(x+1)}{2x(x^2-7x-8)} = \frac{(x+1)(x^2-1)}{2x(x-8)(x+1)} = \frac{(x+1)(x-1)(x+1)}{2x(x-8)(x+1)} = \frac{(x-1)^2}{2x(x-8)}$$

$$7b) x + \frac{1}{x} = 5 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 5^2 \Rightarrow x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} = 25 \Rightarrow x^2 + \frac{1}{x^2} = 25 - 2 \Rightarrow x^2 + \frac{1}{x^2} = 23$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right) \left(x^2 + x \cdot \frac{1}{x} + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} + 1\right) = 5 \cdot (23 - 1) = 5 \cdot 22 = 110$$

$$8) A = \frac{x^3+x}{2x} - \frac{x(x^2-1)}{2x} = \frac{(x-1)(x+1)}{2} \quad , \quad B = \frac{x^2-x-2}{2x-4} = \frac{(x-2)(x+1)}{2(x-2)} = \frac{x+1}{2} \Rightarrow$$

$$\frac{A}{B} = \frac{\frac{(x-1)(x+1)}{2}}{\frac{x+1}{2}} = \frac{\cancel{2}(x-1)\cancel{(x+1)}}{\cancel{2}(x+1)} = x-1 \Rightarrow$$

$$\left(\frac{A}{B}\right)^3 = (x-1)^3 = x^3 - 3x^2 + 3x - 1$$

$$9) f(x) = x^3 - x^2 + 12$$

$$a) f(x-3) = (x-3)^3 - (x-3)^2 + 12 = x^3 - 9x^2 + 27x - 27 - x^2 + 6x - 9 + 12 = x^3 - 10x^2 + 33x - 24$$

$$ii) \text{ a) } \begin{array}{r} x^3 - 10x^2 + 33x - 24 \\ -x^3 + x^2 \\ \hline -9x^2 + 33x - 24 \\ +9x^2 - 9x \\ \hline 24x - 24 \\ -24x + 24 \\ \hline 0 \end{array} \quad \left. \begin{array}{l} x-1 \\ x^2 - 9x + 24 \end{array} \right\}$$

$\hookrightarrow v=0 \Rightarrow$ είναι άρατος του $f(x-3)$

b) άρατος

$x-1=0 \Rightarrow x=1$ και να είναι άρατος του $f(x-3)$ αρκεί να $x=1$ να είναι πηλίκο του. Δηλ. και $x=1 \Rightarrow f(x-3)=0$

και $x=1$ έχουμε $1^3 - 10 \cdot 1^2 + 33 \cdot 1 - 24 = 1 - 10 + 33 - 24 = 0$ Άρα είναι άρατος.

$$10) a) x^2 - 5x + 4 = 0 \Rightarrow (x-4)(x-1) = 0 \Rightarrow x=4 \text{ ή } x=1$$

$$-2x^2 = 3x - 5 \Rightarrow -2x^2 - 3x + 5 = 0 \Rightarrow x_{1,2} = \frac{3 \pm \sqrt{49}}{-4} = \frac{3 \pm 7}{-4} \Rightarrow \begin{array}{l} \nearrow x=1 \\ \searrow x = -\frac{5}{2} \end{array}$$

$$\Delta = 9 + 40 = 49$$

$$\begin{array}{r} 2x^2 + 3x - 5 \\ -2x^2 + 2x \\ \hline 5x - 5 \\ -5x + 5 \\ \hline 0 \end{array} \quad \left. \begin{array}{l} x-1 \\ 2x+5 \end{array} \right\} \text{ άρατος είναι άρατος. άρα } -2x^2 - 3x + 5 = -(2x^2 + 3x - 5) = -(x-1)(2x+5)$$

$$A = \frac{x-1}{x^2-6x+4} = \frac{\cancel{x-1}}{(x-4)(x-1)} = \frac{1}{x-4} \quad B = \frac{-2x^2-3x+5}{x^2-1} = \frac{-(x-1)(2x+5)}{(x-1)(x+1)} = -\frac{2x+5}{x+1}$$

$$8) K = 5 \left[\frac{2x+5}{x+1} \right] + 30 \cdot \frac{1}{x-4} \cdot (x-4) \quad K = 5 \cdot \left[-\frac{2x+5}{x+1} \right] + 30 \cdot \frac{1}{x-4} \cdot (x-4) \Rightarrow$$

$$K = 5 \left[-\frac{2x+5}{x+1} \right] + 30 \Rightarrow K = -5 + 30 \Rightarrow K = 25$$

$$6) \left(\frac{1}{A}\right)^2 = K \Rightarrow (x-4)^2 = 25 \Rightarrow \begin{array}{l} x-4=5 \quad \text{ή} \quad x-4=-5 \\ \Rightarrow x=9 \quad \text{ή} \quad x=-1 \\ \text{δεν είναι} \quad \text{δεν είναι} \end{array}$$

$$11) \begin{array}{r} 2x^2 + 7x - 15 \\ -2x^2 + 3x \\ \hline 10x - 15 \\ -10x + 15 \\ \hline 0 \end{array} \left| \begin{array}{l} 2x-3 \\ x+5 \end{array} \right.$$

Αρα ο άρτος κοσμήτης είναι το $x+5$

$$12) \Delta(x) = \pi(x) \cdot \delta(x) + \nu(x) \Rightarrow$$

$$\Delta(y) = (y^2 - 3y + 5)(2y + 4) + 3 \Rightarrow \Delta(y) = 2y^3 + 4y^2 - 6y^2 - 12y + 10y + 20 + 3 \Rightarrow$$

$$\Delta(y) = 2y^3 - 2y^2 - 2y + 23$$

$$13) (x^2 + 7x + 12) : (x + 3) = \frac{(x+3)(x+4)}{x+3} = x + 4$$

$$(4x^2 - 12x + 9) : (2x - 3) = (2x - 3)^2 = (2x - 3) = 2x - 3$$

$$\frac{x^2 - 6x + 8}{x - 2} = \frac{(x-2)(x-4)}{x-2} = x - 4$$

$$(x^3 + 2x^2 - 13x - 3) : (x - 3) = x^2 + 5x + 2$$

$$(3x^3 + x^2 - 4) : (x - 1) = 3x^2 + 4$$

$$\begin{array}{r} 3x^2 + x - 4 \\ -3x^3 + 3x^2 \\ \hline 4x^2 - 4 \\ -4x^2 + 4 \\ \hline 0 \end{array} \left| \begin{array}{l} x-2 \\ 3x^2-4 \end{array} \right.$$

$$\begin{array}{r} x^3 + 2x^2 - 13x - 3 \\ -x^3 + 3x^2 \\ \hline 5x^2 - 13x - 3 \\ -5x^2 + 15x - 3 \\ \hline 2x - 6 \\ -2x + 6 \\ \hline 0 \end{array} \left| \begin{array}{l} x-3 \\ x^2+5x+2 \end{array} \right.$$

$$14) x - \frac{3}{x} = 4 \Rightarrow \left(x - \frac{3}{x}\right)^2 = 16 \Rightarrow x^2 - 2x \cdot \frac{3}{x} + \frac{9}{x^2} = 16 \Rightarrow x^2 + \frac{9}{x^2} = 16 + 6 \Rightarrow x^2 + \frac{9}{x^2} = 25$$

$$x^3 - \frac{27}{x^3} = \left(x - \frac{3}{x}\right) \cdot \left(x^2 + x \cdot \frac{3}{x} + \frac{9}{x^2}\right) = 4 \cdot (25 + 3) = 4 \cdot 28 = 112$$

$$15) P(x) - \varphi(x) = (3x - 2) - (x^3 + 3x^2 - 5x + 10) = 3x - 2 - x^3 - 3x^2 + 5x - 10 = -x^3 + 5x - 12$$

$$6P(x) = 6(3x - 2) = 18x - 12$$

$$\varphi(-3) = (-3)^3 + 3(-3)^2 - 5(-3) + 10 = -27 + 27 + 15 + 10 = 25$$

$$P[\varphi(-3)] = P(25) = 3 \cdot 25 - 2 = 75 - 2 = 73$$

$$\sigma(x) - 3\varphi(x) = 6 \Rightarrow 3x^3 - 9x^2 - 10x + 6 - 3x^3 - 9x^2 + 15x - 30 = 6 \Rightarrow 5x - 30 = 6 \Rightarrow x = 6$$

$$16) A = (2x - 3)^2 - (x - 1)(x + 1) + (x - 1)^3 \Rightarrow$$

$$A = 4x^2 - 12x + 9 - x^2 + x + x^3 - 3x^2 + 3x - 1 \Rightarrow A = -9x^2 + 9 = 9(1 - x) \quad [-9(x + 1)]$$

$$B = x^2 + x + 1 \Rightarrow A \cdot B = (x^2 + x + 1) \cdot [-9(x - 1)] = -9(x - 1)(x^2 + x + 1) = -9(x^3 - 1) = -9x^3 + 9$$

$$\frac{x^2 - 10x + 25}{x^3 - 5x^2} = \frac{(x-5)^2}{x^2(x-5)} = \frac{(x-5)^2 \cdot x}{x^2(x-5)(x+5)} = \frac{1}{x(x+5)}$$

$$x^2 \cdot (x-5) \neq 0 \quad x \neq 0 \quad x^2 - 5 \neq 0 \Rightarrow (x-5)/(x+5) \neq 0$$

$$\underline{x \neq 0} \quad \text{και} \quad \underline{x \neq 5} \quad \text{και} \quad \underline{x \neq -5}$$

$$17) \frac{A}{B} = \frac{\frac{x^6 y^5}{4}}{-\frac{x^4 y^2}{2}} = -\frac{2x^6 y^5}{4x^4 y^2} = -\frac{1}{2} x^2 y^3 \quad 8x^{4-3} y^{2+1}$$

apa $m-3=2 \Rightarrow m=5$ dan $2n+1=3 \Rightarrow n=1$

$$18) (2x+1)^3 - 2x(3x+1)(3x-1) - (3x-2)^2 =$$

$$= 8x^3 + 12x^2 + 6x + 1 - 2x(9x-1) - (9x^2 - 12x + 4) =$$

$$= 8x^3 + 12x^2 + 6x + 1 - 18x^2 + 2x - 9x^2 + 12x - 4 = 8x^3 - 15x^2 + 20x - 3$$

atau $x=-2 \Rightarrow 8(-2)^3 - 15(-2)^2 + 20(-2) - 3 = 8 \cdot 8 - 15 \cdot 4 - 40 - 3 = 64 - 60 - 40 - 3 = -39$

$$19) x = \frac{1}{y} \Rightarrow A = \left(\frac{5}{y} - y\right)^2 - \left(\frac{5}{y} - 3\right)\left(\frac{5}{y} + 3\right) - (y+2)^2 \Rightarrow$$

$$A = \frac{25}{y^2} - 10 + y^2 - \frac{25}{y^2} + 9 - 9 - 4y - 4 = -4y - 5$$

$$20) 2x + y = -5$$

$$A = (2x+y)^2 + 7xy - x(2-y) + 2x = 4x^2 - 4xy + y^2 + 7xy - 2x + xy + 2x \Rightarrow$$

$$A = 4x^2 + 4xy + y^2 = (2x+y)^2 = (-5)^2 = 25$$

$$21) 2a - b = 7 \quad ab = 10$$

$$(4a^2 + b^2) = (2a - b)^2 + 4ab = 7^2 + 4 \cdot 10 = 49 + 40 = 89$$

$$22) a(a-2) - b(b-2) = a^2 - 2a - b^2 + 2b = (a-b)(a+b) - 2(a-b) = (a-b)(a+b-2)$$

$$x^2 - 6x + 9 - 2bx + 6b = (x-3)^2 - 2b(x-3) = (x-3)(x-3-2b)$$

$$(x-3w)^2 + (x-3w) - 6 = (x-3w+3)(x-3w-2) \quad \text{ws } 7pwwp0..$$

$$x^2 - 6x - 1 - 9y^2 + 4 - 4x = (x^2 - 4x + 4) - (9y^2 + 6y + 1) = (x-2)^2 - (3y+1)^2 =$$

$$= (x-2-3y-1)(x-2+3y+1) = (x-3y-3)(x+3y-1)$$

$$16x^4 - 81y^4 = (4x^2 - 9y^2)(4x^2 + 9y^2) = (2x-3y)(2x+3y)(4x^2 + 9y^2)$$

$$3p^2 - 3w^2 - w^2 - 2pw - p^2 = 3(p^2 - w^2) - (w^2 + 2pw + p^2) = 3(p-w)(p+w) - (w+p)^2 =$$

$$= (w+p)(3p-3w-w-p) = (w+p)(2p-2w) =$$

$$= 2(w+p)(p-w)$$

$$a^2(a-5) + (a-5)(3a-2) - 25 + a^2 =$$

$$= a^2(a-5) + (a-5)(3a-2) + (a^2 - 25) =$$

$$= a^2(a-5) + (a-5)(3a-2) + (a-5)(a+5) =$$

$$= (a-5)(a^2 + 3a - 2 + a + 5) = (a-5)(a^2 + 4a + 3) =$$

$$= (a-5)(a+3)(a+1)$$

$$x^2 - 6xy + 9y^2 - 4w^2 = (x-3)^2 - 4w^2 = (x-3)^2 - (2w)^2 =$$

$$= (x-3-2w)(x-3+2w)$$

$$(x^2 - 6x + 3)^2 - (x-9)^2 = (x^2 - 6x + 3 - x + 9)(x^2 - 6x + 3 + x - 9) =$$

$$= (x^2 - 7x + 12)(x^2 - 5x + 6) = (x-4)(x-3)(x-2)(x-3) =$$

$$= (x-3)^2(x-4)(x-2)$$

$$4(x-1) + 9x^2(1-x) = (x-1) - 9x^2(x-1) = (x-1)(1-9x^2) = (x-1)(1-3x)(1+3x)$$

$$23) A = 2x^3 + 6xy^2 - 2y^3 - 6x^2y = 2(x^3 - y^3) - 6xy(x-y) =$$

$$= 2(x-y)(x^2 + xy + y^2) - 6xy(x-y) =$$

$$= 2(x-y)(x^2 + xy + y^2 - 3xy) = 2(x-y)(x^2 - 2xy + y^2)$$

$$= 2(x-y)(x-y)^2 = 2(x-y)^3 = 2(101-99)^3 = 2 \cdot 2^3 = 16$$

$$24) \frac{\frac{x}{y} - \frac{9y}{x}}{\frac{x^2 - 6x}{y^2} + 9} = \frac{\frac{x^2 - 9y^2}{xy}}{\frac{x^2 - 6x + 9y^2}{y^2}} = \frac{\cancel{(x-3y)}(x+3y) \cdot y^2}{x \cdot y \cdot (x-3y)^2} = \frac{y(x+3y)}{x(x-3y)}$$

$$\frac{\frac{x^2 - 16}{x^2 + 3x - 4}}{\frac{x^2 - 4x}{x^2}} = \frac{\cancel{(x-4)}(x+4) \cdot x}{\cancel{(x+4)}(x-1) \cdot x \cdot \cancel{(x-4)}} = \frac{x}{x-1}$$