Λύσεις εργασία 2

1)

Μονώνυμο	Συντελεστής	Κ. Μέρος
$5a^3$	5	a^3
$\frac{3x\psi}{4}$	$3/4$	$x\psi$
$-a^3b^4$	-1	a^3b^4

2) $-4x^3\psi^5\omega$

α) $3^{ου}$ βαθμού
β) $9^{ου}$ βαθμού
γ) $x^3\psi^5\omega^1$
δ) -4
ε) $7x^3\psi^5\omega$

3)
α) Λάθος
β) Σωστό
γ) Λάθος
δ) Λάθος
ε) Λάθος
στ) Λάθος.

4) $\Delta = \delta \cdot \pi + \upsilon = (3x+1)(x^2+2x-4)+7$
$\Delta = 3x^3+6x^2-12x+x^2+2x-4+7$
$\Delta = 3x^3+7x^2-10x+3$

5)
α → iii
β → iv
γ → v
δ → vi

6)

α) $(3x^2-x+4)+(x^2+6x-3)=$

$=3x^2-x+4+x^2+6x-3=4x^2+5x+1$

β) $(\psi^2+2\psi-3)-(-3\psi^2+5)=$

$=\psi^2+2\psi-3+3\psi^2-5=4\psi^2+2\psi-8$

γ) $(-3x^2\omega)\cdot(7x\psi)=-21x^3\omega\psi$

δ) $a(a-5)=a^2-5a$

ε) $3\psi(\psi^2-2\psi+4)=3\psi^3-6\psi^2+12\psi$

στ) $(x+4)(x-2)=x^2-2x+4x-8=x^2+2x-8$

ζ) $(12x^4\psi^3-16x^3\psi^5+20\psi^2x^5a):(-4x^3\psi^3)=$

$=-3x+4\psi^2-5\psi^{-1}x^2a$

7) α) $-6ab$ και $\frac{12}{\lambda}ab$

αντίθετα αρκεί $\frac{12}{\lambda}=6 \Rightarrow 6\lambda=12 \Rightarrow \frac{6\lambda}{6}=\frac{12}{6}$

$\boxed{\lambda=2}$

β) $-7b^{2k+1}\cdot\gamma^{k-2}$

για να είναι 8ου βαθμού

αρκεί $2k+1+k-2=8$

$3k-1=8 \Rightarrow 3k=8+1 \Rightarrow \frac{3k}{3}=\frac{9}{3} \Rightarrow$

$\boxed{k=3}$

8) α) $(3x+2)^2-(3x-2)^2=24x$

α΄ μέλος: $(3x+2)^2-(3x-2)^2=$

$=(3x+2)(3x+2)-(3x-2)(3x-2)=$

$=9x^2+6x+6x+4-(9x^2-6x-6x+4)=$

$=\cancel{9x^2}+12x+\cancel{4}-\cancel{9x^2}+6x+6x-\cancel{4}=24x$ β΄ μέλος

β) $(x+2)^2-(3x^2+5x-2)=(2x-1)-(2x-1)(x+2)+5$

α΄ μέλος: $(x+2)^2-(3x^2+5x-2)=$

$=(x+2)(x+2)-(3x^2+5x-2)=$

$=x^2+2x+2x+4-3x^2-5x+2=$

$=-2x^2-x+6$

β΄ μέλος: $(2x-1)-(2x-1)(x+2)+5=$

$=2x-1-(2x^2+4x-x-2)+5$

$=\cancel{2x}-1-2x^2-\cancel{4x}+x+2+5$

$=-2x^2-x+6$

Άρα α΄ μέλος = β΄ μέλος

9) α) $P(x)+q(x)=$

$6x^2-1+3x+x-2=6x^2+4x-3$

β) $q(x)\cdot r(x)=(x-2)\cdot(3x^2+x-5)=$

$=3x^3+x^2-5x-6x^2-2x+10=$

$=3x^3-5x^2-7x+10$

γ) $P(3)-q(10)+r(1)=62-8-1=62-9=\underline{\underline{53}}$

$P(3)=6\cdot3^2-1+3\cdot3=6\cdot9-1+9=54+9-1=62$

$q(10)=10-2=8$

$r(1)=3\cdot1^2+1-5=3+1-5=-1$

δ) Πρέπει $p(x) : q(x)$ να έχουν υπόλοιπο 0

$$
\begin{array}{r|l}
6x^2+3x-1 & x-2 \\ \cline{2-2}
-6x^2+12x & 6x+15 \\ \cline{1-1}
15x-1 & \\
-15x+30 & \\ \cline{1-1}
+29 &
\end{array}
$$

Άρα το $q(x)$ δεν είναι παράγοντας του $p(x)$ γιατί έχει $υ=29$

ε) $p(x)-2r(x)+5\cdot q(x)=11$

$6x^2+3x-1-2(3x^2+x-5)+5(x-2)=11$

$\cancel{6x^2}+3x-1-\cancel{6x^2}-2x+\cancel{10}+5x-\cancel{10}-11=0$

$6x-12=0 \Rightarrow 6x=12 \Leftrightarrow x=\frac{12}{6} \Leftrightarrow \boxed{x=2}$

10) α) $φ(x)+σ(x)-ρ(x)=(4x^2-8x+3)+(2x^2+9)-(2x-1)=$

$=4x^2-8x+3+2x^2+9-2x+1=6x^2-10x+13$

β) $φ(x):ρ(x)$

$$
\begin{array}{r|l}
4x^2-8x+3 & 2x-1 \\ \cline{2-2}
-4x^2+2x & 2x-3 \\ \cline{1-1}
-6x+3 & \\
+6x-3 & \\ \cline{1-1}
= &
\end{array}
$$

γ) $\sqrt{φ(-2)+2ρ(4)}-\sqrt[3]{σ(3)}=$

$=\sqrt{35+2\cdot 7}-\sqrt[3]{27}=\sqrt{35+14}-3$

$=\sqrt{49}-3=7-3=\underline{\underline{4}}$

$φ(-2)=4(-2)^2-8(-2)+3$
$=4\cdot 4+16+3=16+16+3=35$

$ρ(4)=2\cdot 4-1=8-1=7$

$σ(3)=2(3)^2+9=2\cdot 9+9=27$

δ) $[p(x)]^2 - 5p(x) = 2\phi(x) - 2x \cdot p(x)$

α' μέλος $[p(x)]^2 - 5p(x) = (2x-1)^2 - 5(2x-1) =$

$= (2x-1)(2x-1) - 5(2x-1) =$

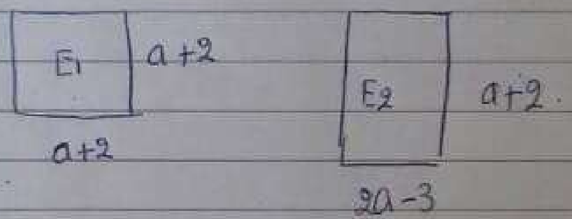
$= 4x^2 - 2x - 2x + 1 - 10x + 5 = 4x^2 - 14x + 6$

β' μέλος: $2\phi(x) - 2x \cdot p(x) = 2(4x^2 - 8x + 3) - 2x(2x-1) =$

$= 8x^2 - 16x + 6 - 4x^2 + 2x = 4x^2 - 14x + 6$

Άρα α' μέλος = β' μέλος.

11)



$E_{ολ} = E_1 + E_2$

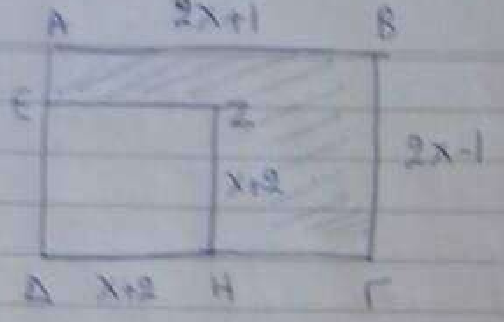
$E_{ολ} = a^2 + 4a + 4 + 2a^2 + a - 6$

$E_{ολ} = 3a^2 + 5a - 2$

$E_1 = (a+2)(a+2)$
$= a^2 + 2a + 2a + 4$
$= a^2 + 4a + 4$

$E_2 = (2a-3)(a+2)$
$= 2a^2 + 4a - 3a - 6$
$= 2a^2 + a - 6$

12)



$E_{εζη} = (x+2)(x+2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4$

$E_{ορ} = (2x+1)(2x-1) = 4x^2 - \cancel{2x} + \cancel{2x} - 1 = 4x^2 - 1$

$E_{σκιασ} = E_{ορ} - E_{τετ} = 4x^2 - 1 - (x^2 + 4x + 4)$

$= 4x^2 - 1 - x^2 - 4x - 4 = 3x^2 - 4x - 5$

$Π_{σκιασ} = ΑΒ + ΒΓ + ΓΗ + ΗΖ + ΖΕ + ΕΑ$

$Π_{σκιασ} = 2x + \cancel{1} + 2x - \cancel{1} + x - 1 + x + 2 + x + 2 + x - 3$

$Π_{σκιασ} = 8x$

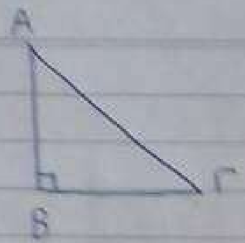
ii.

$Π_{ορθ} = 2(2x+1) + 2(2x-1) =$
$= 4x + \cancel{2} + 4x - \cancel{2} = 8x$

$ΗΓ = ΓΔ - ΔΗ = 2x + 1 - (x+2) = 2x + 1 - x - 2 = x - 1$

$ΑΕ = 2x - 1 - (x+2) = 2x - 1 - x - 2 = x - 3$

13)



$A\Gamma > AB > B\Gamma$

α) i) $A\Gamma = -5x + 6x(x-1) - (2x-5)(3x+1) =$

$= -5x + 6x^2 - 6x - (6x^2 + 2x - 15x - 5) =$

$= -5x + 6x^2 - 6x - 6x^2 - 2x + 15x + 5 =$

$= 2x + 5$

ii) $AB = 2\sqrt{10x}$, $\quad B\Gamma = 2x - 5$, $\quad A\Gamma = 2x + 5$

$(A\Gamma)^2 = (2x+5)^2 = (2x+5)(2x+5)$

$= 4x^2 + 10x + 10x + 25 =$

$= 4x^2 + 20x + 25$

$(AB)^2 + (B\Gamma)^2 = (2\sqrt{10x})^2 + (2x-5)^2 =$

$= 2^2 \cdot (\sqrt{10x})^2 + (2x-5)(2x-5) =$

$= 4 \cdot (10x) + 4x^2 - 10x - 10x + 25 =$

$= 40x + 4x^2 - 20x + 25$

$= 4x^2 + 20x + 25$

$\Rightarrow$

$\Rightarrow (A\Gamma)^2 = (AB)^2 + (B\Gamma)^2$ άρα ισχύει το ΠΘ

$\Rightarrow$ ΑΒΓ ορθογώνιο $\hat{B} = 90^\circ$

β)

$E = \frac{\beta \cdot \upsilon}{2} \Rightarrow \frac{2x^2 + 9x + 10}{2} = \frac{(2x+5) \cdot \upsilon}{2}$

$2x^2 + 9x + 10 = (2x+5)\upsilon \Rightarrow \upsilon = (2x^2 + 9x + 10) : (2x+5)$

$$\begin{array}{r|l} \cancel{2x^2} + 9x + 10 & 2x + 5 \\ \hline -\cancel{2x^2} - 5x & x + 2 \\ \hline \cancel{4x} + \cancel{10} & \\ -\cancel{4x} - \cancel{10} & \\ \hline = & \end{array}$$